Transient Energy Transfer by Conduction and Radiation in Nongray Gases

S. N. Tiwari* and D. J. Singh†
Old Dominion University, Norfolk, Virginia
and
A. Kumar‡
NASA Langley Research Center, Hampton, Virginia

A general formulation is presented to investigate the transient radiative interaction in nongray absorbing-emitting species between two parallel plates. Depending on the desired sophistication and accuracy, any nongray absorption model from the line-by-line models to the wide-band model correlations can be employed in the formulation to investigate the radiative interaction. Special attention is directed to investigate the radiative interaction in a system initially at a uniform reference temperature when the temperature of the bottom plate is reduced suddenly to a lower but constant temperature. The interaction is considered for the case of radiative equilibrium as well as for combined radiation and conduction. General as well as limiting forms of the governing equations are presented, and solutions are obtained numerically by employing the method of variation of parameters. Specific results are obtained for CO, CO_2, H_2O, and OH. The information on species H_2O and OH is of special interest for the proposed scramjet engine application. The results demonstrate the relative ability of different species for radiative interactions.

Nomenclature

A	= band absorptance = $A(u,\beta)$, cm^{-1}
A_o	= bandwidth parameter, cm^{-1}
C_o	= correlation parameter, atm^{-1} − cm^{-1}
C_p	= specific heat at constant pressure, kJ/kg-K = erg/gm-K
e_ω	= Planck's function, (W-cm^{-2})/cm^{-1}
e_{ω_o}	= Planck's function evaluated at wavenumber ω_o
e_1,e_2	= emissive power of surfaces with temperatures T_1 and T_2, W-cm^{-2}
H_{1i},H_1	= gas property for the large path length limit, W/cm^2-K
k	= thermal conductivity, erg/cm-s-K
K_{1i},K_1	= gas property for the optically thin limit, W/(atm-cm^3-K)
L	= distance between plates, cm
M_{1i},M_1	= large path length parameter, nondimensional
N	= optically thin radiation-conduction parameter = N_1/R, nondimensional
N_{1i},N_1	= optically thin parameter, nondimensional
p	= pressure, atm
q	= conduction plus radiation heat flux = $q_c + q_R$, W/cm^2
q_c	= conduction heat flux, W/cm^2
q_R	= total radiative heat flux, W/cm^2
Q	= nondimensional radiative heat flux
$\bar{Q}$	= nondimensional conduction plus radiation heat flux
$q_{R\omega}$	= spectral radiation heat flux, (W-cm^{-2})/cm^{-1}
R	= nondimensional transient conduction parameter
S	= integrated intensity of a wide band, atm^{-1}-cm^{-2}
t	= time, s (also used as t^*)
t_m	= characteristic time, s
t^*	= nondimensional time = t/t_m
T	= temperature, K
T_1,T_2	= wall temperature, K; $T_1 = T_w$
u	= nondimensional coordinate = SPy/A_o
u_o	= nondimensional path length = SPL/A_o
y	= transverse coordinate, cm
θ	= nondimensional temperature
κ_ω	= spectral absorption coefficient, cm^{-1}
ξ	= nondimensional coordinate = $y/L = u/u_o$
ρ	= density, kg/m^3
σ	= Stefan-Boltzmann constant, erg/(s-cm^2-K^4)
ω	= wavenumber, cm^{-1}
ω_o	= wavenumber at the band center, cm^{-1}

Introduction

THE field of radiative energy transfer in gaseous systems is getting ever increasing attention recently because of its applications in the areas of the Earth's radiation budget studies and climate modeling, fire and combustion research, entry and re-entry phenomena, hypersonic propulsion, and defense-oriented research. In most studies involving combined mass, momentum, and energy transfer, however, the radiative transfer formulation has been coupled mainly with the steady processes,[1-11] and the interaction of radiation in transient processes has received very little attention. Yet the transient approach appears to be the logical way of formulating a problem in a general sense for elegant numerical and computational solutions. The steady-state solutions can be obtained as limiting solutions for large times.

A few studies available on radiative interactions reveal that the transient behavior of a physical system can be influenced significantly in the presence of radiation.[12-17] Lick investigated the transient energy transfer by radiation and conduction through a semifinite medium.[12] A kernal substitution technique was used to obtain analytic solutions and display the main features and parameters of the problem. Steady and transient heat transfer in conducting and radiating planar and cylindri-

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*Eminent Professor, Department of Mechanical Engineering and Mechanics. Associate Fellow AIAA.

†Graduate Research Assistant, Department of Mechanical Engineering and Mechanics. Student Member AIAA.

‡Head, Computational Methods Branch, High-Speed Aerodynamics Division. Associate Fellow AIAA.

cal media were analyzed in Refs. 13 and 14 according to the differential formulation. The analyses, based essentially on the gray formulation, provide some qualitative insight into the effect of absorption and emission on the transient temperature distribution in the gas. Doornink and Hering[15] studied the transient radiative transfer in a stationary plane layer of a nonconducting medium bounded by black walls. A rectangular Milne-Eddington-type relation was used to describe the frequency dependence of the absorption coefficient. It was found that the initial cooling of the layer at a uniform temperature is strongly dependent on the absorption coefficient model employed. Larson and Viskanta[16] investigated the problem of transient combined laminar free convection and radiation in a rectangular enclosure. It was demonstrated that the radiation dominates the heat transfer in the enclosure and alters the convective flow patterns significantly. The transient heat exchange between a radiating plate and a high-temperature gas flow was investigated by Melnikov and Sukhovich.[17] Only the radiative interaction from the plate was considered; the gas was treated as a nonparticipating medium. It was proved that the surface temperature is a function of time and of longitudinal coordinate. Some other works on transient radiation and related areas are available in Refs. 18–23.

The goal of this research is to include the nongray radiative formulation in the general unsteady governing equations and provide the step-by-step analysis and solution procedure for several realistic problems. The specific objective of the present study is to investigate the interaction of nongray radiation in transient transfer processes in a general sense. Attention, however, will be directed first to a simple problem of the transient radiative exchange between two parallel plates. In subsequent studies, the present analysis and numerical techniques will be extended to include the flow of homogeneous, nonhomogeneous, and chemically reacting species in one- and multidimensional systems.

Basic Theoretical Formulation

The physical model considered for the present study is the transient energy transfer by radiation in absorbing-emitting gases bounded by two parallel gray plates (Fig. 1). In general, T_1 and T_2 can be a function of time and position, and there may exist an initial temperature distribution in the gas. It is assumed that the radiative energy transfer in the axial direction is negligible in comparison to that in the normal direction.

For a radiation participating medium, the equations expressing conservation of mass and momentum remain unaltered, while the conservation of energy in general is expressed as[1]

$$\rho c_p \frac{\mathrm{D}T}{\mathrm{D}t} = \mathrm{div}(k \text{ grad } T) + \beta T \frac{\mathrm{D}P}{\mathrm{D}t} + \mu\phi - \mathrm{div}\, q_R \tag{1}$$

where μ is dynamic viscosity, β is the coefficient of thermal expansion of the fluid, and ϕ is the Rayleigh dissipation function. For a semi-infinite medium capable of transferring energy only by radiation and conduction, Eq. (1) reduces to

$$\rho c_p \frac{\partial T}{\partial t} = -\frac{\partial q}{\partial y} \tag{2}$$

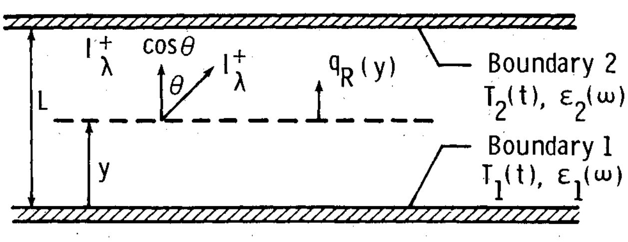

Fig. 1 Physical model and coordinate system.

where q is the sum of the conductive heat flux $q_c = -k(\partial T/\partial y)$ and the radiative flux q_R. For the physical model where radiation is the only mode of energy transfer, Eq. (2) reduces to a simplified form.

For many engineering and astrophysical applications, the radiative transfer equations are formulated for one-dimensional planar systems. For diffuse nonreflecting boundaries and in the absence of scattering, the expression for the total radiative flux is given, for an n-band gaseous system, by[1,8,24–26]

$$\begin{aligned} q_R(y) = e_1 - e_2 + \tfrac{3}{2} \sum_{i=1}^{n} \int_{\Delta\omega_i} \Bigg\{ \int_0^y & F_{1\omega_i}(z)\kappa_{\omega_i} \\ \times \exp[-\tfrac{3}{2}\kappa_{\omega_i}(y - z)]\, \mathrm{d}z - \int_y^L & F_{2\omega_i}(z)\kappa_{\omega_i} \\ \times \exp[-\tfrac{3}{2}\kappa_{\omega_i}(z - y)]\, \mathrm{d}z \Bigg\} & \mathrm{d}\omega_i \end{aligned} \tag{3}$$

where

$$F_{1\omega_i}(z) = e_{\omega_i}(z) - e_{1\omega_i}; \qquad F_{2\omega_i}(z) = e_{\omega_i}(z) - e_{2\omega_i}$$

Equation (3) is in proper form for obtaining the nongray solutions of molecular species. In fact, this is an ideal equation for the line-by-line and narrow-band model formulations. However, in order to be able to use the wide-band models and correlations, Eq. (3) is transformed in terms of the correlation quantities as[1,7–11,24–26]

$$\begin{aligned} q_R(\xi) = e_1 - e_2 + \tfrac{3}{2} \sum_{i=1}^{n} A_{oi}u_{oi} \Bigg\{ \int_0^\xi F_{1\omega_i}(\xi')\bar{A}'_i[\tfrac{3}{2}u_{oi}(\xi - \xi')]\, \mathrm{d}\xi' \\ - \int_\xi^1 F_{2\omega_i}(\xi')\bar{A}'_i[\tfrac{3}{2}u_{oi}(\xi' - \xi)]\, \mathrm{d}\xi' \Bigg\} \end{aligned} \tag{4}$$

where

$$\xi = u/u_o = y/L; \qquad \xi' = u'/u_o = z/L; \qquad \bar{A} = A/A_o;$$

$$u = (S/A_o)Py; \qquad u_o = (S/A_o)PL; \qquad PS = \int_{\Delta\omega} \kappa_\omega\, \mathrm{d}\omega$$

It should be noted that $F_{1\omega_i}$ and $F_{2\omega_i}$ in Eq. (4) represent the values at the center of the ith band and $\bar{A}'(u)$ denotes the derivative of $\bar{A}(u)$ with respect to u. Upon performing the integration by parts, Eq. (4) and its derivative can be expressed in various forms (see Refs. 24–26).

By defining $\phi(\xi,t) = T(\xi,t)/T_o$ with T_o representing some constant reference temperature, Eqs. (2) and (4) can be combined to yield the energy equation (for the general case of simultaneous conduction and radiation) in nondimensional form as[25,26]

$$\begin{aligned} \partial\phi(\xi,t)/\partial t = R\frac{\partial^2\phi}{\partial\xi^2} - \tfrac{3}{2}\sum_{i=1}^{n} \Bigg\{ \int_0^\xi \psi_{\omega_i}(\xi',t)\bar{A}'_i[\tfrac{3}{2}u_{oi}(\xi - \xi')]\, \mathrm{d}\xi' \\ - \int_\xi^1 \psi_{\omega_i}(\xi',t)\bar{A}'_i[\tfrac{3}{2}u_{oi}(\xi' - \xi)]\, \mathrm{d}\xi' \Bigg\} \end{aligned} \tag{5}$$

where

$$R = kt_m/(\rho C_p L^2)$$

$$\psi_{\omega_i}(\xi,t) = \{PS_i(T)[\partial e_{\omega_i}(\xi,t)/\partial\xi]/(\rho C_p T_o/t_m)\}$$

The time t in Eq. (5) is defined as $t^* = t/t_m$ with t_m representing some characteristic time scale of the physical problem; however, for the sake of convenience, the asterisk is left out here as well as in further developments. From the definitions of $\phi(\xi,t)$ and $\psi_{\omega_i}(\xi,t)$, it should be noted that Eq. (5) is a nonlinear equation in $T(\xi,t)$. Equation (5), therefore, represents a general case of the transient energy transfer by radiation and conduction between two semi-infinite parallel plates. The nondimensional parameter R in Eq. (5) is analogous to the Fourier number. For $R = 0$, Eq. (5) reduces to the case of pure transient radiative energy transfer. The initial and boundary conditions for Eq. (5) will depend on the conditions of the specific physical problem.

As a special case, it is assumed that the entire system is initially at the fixed (reference) temperature T_o. For all time, the temperature of the upper plate is maintained at the constant temperature equal to the reference temperature, i.e., $T_2 = T_o$. The temperature of the lower plate is suddenly decreased to a lower but constant value, i.e., $T_1 < T_2$. The problem, therefore, is to investigate the transient cooling rate of the gas for a step change in temperature of the lower plate.

In many radiative transfer analyses, it is often convenient (although not essential) to employ the relation for the linearized radiation as

$$e_{\omega_i}(T) - e_{\omega_i}(T_w) \simeq (\mathrm{d}e_{\omega_i}/\mathrm{d}T)_{T_w}(T - T_w) \tag{6}$$

where again the subscript i refers to the ith band such that ω_i is the wavenumber location of the band and T_w represents the temperature of the reference wall, which could be either T_1 or T_2. For the special case considered, since we are interested in investigating the transient behavior of the gas because of a step change in temperature of the lower plate, T_w is taken to be equal to T_1. It should be pointed out that for a single-band gas, the linearization is not required because the temperature distribution can be obtained from Eq. (5) and the radiative heat flux can be calculated from Eq. (4). However, for the case of multiband gases and for systems involving mixtures of gases, it is convenient to employ the linearization procedure in order to use the information on band model correlations. The following definitions are useful in expressing the governing equations in linearized forms:

$$\theta = (T - T_1)/(T_2 - T_1) \tag{7a}$$

$$N_{1i} = (Pt_m/\rho c_p)K_{1i}, \qquad K_{1i} = S_i(T)(\mathrm{d}e_{\omega_i}/\mathrm{d}T)_{T_1} \tag{7b}$$

$$N_1 = (Pt_m/\rho c_p)K_1, \qquad K_1 = \sum_{i=1}^{n} K_{1i} \tag{7c}$$

$$M_{1i} = (t_m/L\rho c_p)H_{1i}, \qquad H_{1i} = A_{oi}(T)(\mathrm{d}e_{\omega_i}/\mathrm{d}T)_{T_1} \tag{7d}$$

$$M_1 = (t_m/L\rho c_p)H_1, \qquad H_1 = \sum_{i=1}^{n} H_{1i} \tag{7e}$$

$$M_{1i}u_{oi} = N_{1i}, \qquad u_{oi}H_{1i} = PLK_{1i} \tag{7f}$$

where H_1, K_1, N_1, and M_1 represent the values of H, K, N, and M evaluated at the temperature T_1. As explained in Refs. 1 and 8, these quantities represent the properties of the gaseous medium.

The initial and boundary conditions for the physical problem considered are

$$\theta(\xi,0) = 1; \qquad \theta(0,t) = 0; \qquad \theta(1,t) = 1 \tag{8}$$

It is important to note that the boundary conditions given in Eq. (8) are only applicable to the case of simultaneous conduction and radiation energy transfer. The cases of transient radiation and radiation with conduction are treated separately in the following sections.

Transient Radiation

By employing the definitions of Eqs. (6) and (7), Eq. (5) is transformed to obtain a convenient form of the energy equation as[24–26]

$$-\frac{\partial\theta(\xi,t)}{\partial t} = \frac{3}{2}\sum_{i=1}^{n} N_{1i}\left\{\int_0^{\xi} \frac{\partial\theta(\xi',t)}{\partial\xi'}\bar{A}_i'[\tfrac{3}{2}u_{oi}(\xi - \xi')]\,\mathrm{d}\xi' - \int_{\xi}^{1} \frac{\partial\theta(\xi',t)}{\partial\xi'}\bar{A}_i'[\tfrac{3}{2}u_{oi}(\xi' - \xi)]\,\mathrm{d}\xi'\right\} \tag{9}$$

The parameters in Eq. (9) are N_1 and u_o. For a given gas, the parameters are the gas pressure and the temperature of the lower wall. The boundary conditions given in Eq. (8) are not applicable to Eq. (9) because this equation does not require a boundary condition. Thus, in this case, the temperature of the medium adjacent to a surface differs from the surface temperature. This is because the temperature of the medium adjacent to a surface is affected not only by the surface but also by all other volume elements and surfaces. The radiation slip method is a means of accounting for such temperature jumps, and this is discussed in Ref. 1.

For the case of transient radiative interaction, the nondimensional radiative heat flux is defined by

$$Q(\xi,t) = q_R(\xi,t)/[e_1(t) - e_2(t)] \tag{10}$$

By employing the definitions of Eqs. (6), (7), and (10), two relations for the radiative flux are obtained from Eq. (4) as[24–26]

$$Q(\xi,t) = 1 - (3/8\sigma T_1^3)\sum_{i=1}^{n} u_{oi}H_{1i}\left\{\int_0^{\xi} \theta(\xi',t)\bar{A}_i'[\tfrac{3}{2}u_{oi}(\xi - \xi')]\,\mathrm{d}\xi' + \int_{\xi}^{1} [1 - \theta(\xi',t)]\bar{A}_i'[\tfrac{3}{2}u_{oi}(\xi' - \xi)]\,\mathrm{d}\xi'\right\} \tag{11a}$$

and

$$Q(\xi,t) = 1 - (1/4\sigma T_1^3)\sum_{i=1}^{n} H_{1i}\left\{\int_0^{\xi} \frac{\partial\theta(\xi',t)}{\partial\xi'}\bar{A}_i[\tfrac{3}{2}u_{oi}(\xi - \xi')]\,\mathrm{d}\xi' - \int_{\xi}^{1} \frac{\partial\theta(\xi',t)}{\partial\xi'}\bar{A}_i[\tfrac{3}{2}u_{oi}(\xi' - \xi)]\,\mathrm{d}\xi'\right\} \tag{11b}$$

Equation (11a) is a convenient form for the optically thin and general solutions while Eq. (11b) is useful for solutions in the large path length limit. Once the solutions for $\theta(\xi,t)$ are known from the energy equation, the appropriate relations for the heat flux can be obtained from Eqs. (11). It should be noted that the quantity $H_1/(\sigma T_1^3)$ in Eqs. (11) is nondimensional.

Radiation and Conduction

For this case, where conduction heat transfer takes place simultaneously with radiative transfer, the energy equation is given by Eq. (5). Thus, a combination of Eqs. (5–7) results in

$$\frac{\partial\theta}{\partial t} = R\frac{\partial^2\theta}{\partial\xi^2} - \frac{3}{2}\sum_{i=1}^{n} N_{1i}\left\{\int_0^{\xi} \frac{\partial\theta(\xi',t)}{\partial\xi'}\bar{A}_i'[\tfrac{3}{2}u_{oi}(\xi - \xi')]\,\mathrm{d}\xi' - \int_{\xi}^{1} \frac{\partial\theta(\xi',t)}{\partial\xi'}\bar{A}_1'[\tfrac{3}{2}u_{oi}(\xi' - \xi)]\,\mathrm{d}\xi'\right\} \tag{12}$$

Since the presence of conduction implies continuity of temperatures at the boundaries, the boundary conditions for Eq. (12) are those given in Eq. (8). The quantity N_1/R can be expressed as $N = N_1/R = (PL^2/k)K_1$. The nondimensional parameter N denotes the relative importance of radiation vs conduction in the gas. For particular values of P and L, it is actually the dimensional gas property $N_1/(RPL^2) = K_1/k$ that represents the relative importance of radiation vs conduction. For the case of no radiation, Eq. (12) reduces to a simplified form for which transient solutions are available in the literature.

In the case of simultaneous conduction and radiation heat transfer, the nondimensional heat flux is defined as

$$Q = [q_c(\xi,t) + q_R(\xi,t)]/[e_1(t) - e_2(t)] \tag{13}$$

Alternately, this can be expressed as

$$\bar{Q} = C(\partial\theta/\partial\xi) + Q \tag{14}$$

where

$$C = k(T_1 - T_2)/\{L[e_1(t) - e_2(t)]\}$$

The expression for Q in Eq. (14) is obtained from either Eq. (11a) or Eq. (11b). It should be noted that the relation for θ needed in Eq. (14) comes from the solution of Eq. (12).

Method of Solutions

The solution procedures for the radiative equilibrium and radiation with conduction cases are available in Ref. 25. For the sake of brevity, only the solution procedure for the radiative equilibrium case is given here.

For the general case of radiative equilibrium, the temperature distribution is obtained from the solution of the energy equation, Eq. (9). Once $\theta(\xi,t)$ is known, the radiative heat flux is calculated by using the appropriate form of Eq. (11). Before discussing the solution procedure for the general case, however, it is desirable to obtain the limiting forms of Eqs. (9) and (11) in the optically thin and large path length limits and investigate the solutions of resulting equations.

Optically Thin Limit

In the optically thin limit, $\bar{A}(u) = u$ and $\bar{A}'(u) = 1$, and therefore, Eq. (9) reduces to[1,8,24]

$$\frac{\partial\theta(\xi,t)}{\partial t} + 3N_1\theta(\xi,t) - \tfrac{3}{2}N_1 = 0 \tag{15a}$$

From an examination of Eq. (15a) along with the definitions given in Eq. (7), it is evident that in the optically thin limit, the temperature distribution in the medium is independent of the ξ coordinate for the case of pure radiative exchange. This is a characteristic of the optically thin radiation in the absence of other modes of energy transfer. Thus, Eq. (15a) can be written as

$$\frac{d\theta(t)}{dt} + 3N_1\theta(t) - \tfrac{3}{2}N_1 = 0; \qquad \theta(\xi,0) = 1 \tag{15b}$$

Since gas properties are evaluated at known reference conditions, N_1 is essentially constant, and the solution of Eq. (15b) is found to be

$$\theta(t) = \tfrac{1}{2}[1 + \exp(-3N_1t)] \tag{16}$$

In the optically thin limit, both forms of Eq. (11) yield the same final relation for the radiative flux as[24]

$$Q(\xi,t) = 1 - [3/(8\sigma T_1^3)](PLK_1)[(1-\xi) + (2\xi - 1)\theta(t)] \tag{17}$$

It should be pointed out that in Eq. (17), the quantity $(PLK_1/\sigma T_1^3)$ is nondimensional. The relation for $\theta(t)$ in Eq. (17) is obtained from Eq. (16). Thus, evaluation of the temperature distribution and radiative heat flux in the optically thin limit does not require numerical solutions.

Large Path Length Limit

In the large path length limit (i.e., for $u_{oi} \gg 1$ for each band), one has $\bar{A}(u) = \ell n(u)$, $\bar{A}'(u) = 1/u$, and $\bar{A}''(u) = -1/u^2$. Thus, in this limit, Eq. (9) reduces to[1,8,24-26]

$$\frac{\partial\theta(\xi,t)}{\partial t} = -M_1\int_0^1 \frac{\partial\theta(\xi',t)}{\partial\xi'}\frac{d\xi'}{(\xi-\xi')} \tag{18}$$

An analytical solution of Eq. (18) may be possible, but numerical solution can be obtained quite easily.

In the large path length limit, Eqs. (11a) and (11b) reduce, respectively, to

$$Q(\xi,t) = 1 - (1/\sigma 4T_1^3)H_1\left[\int_0^1 \theta(\xi',t)\frac{d\xi'}{(\xi-\xi')} - \int_\xi^1 \frac{d\xi'}{(\xi-\xi')}\right] \tag{19a}$$

and

$$Q(\xi,t) = 1 - (1/4\sigma T_1^3)\sum_{i=1}^{n} H_{1i}\left\{\int_0^\xi \frac{\partial\theta(\xi',t)}{\partial\xi'}\ell n[\tfrac{3}{2}u_{oi}(\xi-\xi')]\,d\xi' + \int_\xi^1 \frac{\partial\theta(\xi',t)}{\partial\xi'}\ell n[\tfrac{3}{2}u_{oi}(\xi'-\xi)]\,d\xi'\right\} \tag{19b}$$

The expressions for dimensionless radiative heat flux from or to the wall are obtained by setting $\xi = 0$ in Eqs. (19).

Numerical Solutions of Governing Equations

The solutions of Eqs. (9) and (18) are obtained numerically by employing the method of variation of parameters. For this, a polynomial form for $\theta(\xi,t)$ is assumed in powers of ξ with time-dependent coefficients as

$$\theta(\xi,t) = \sum_{m=0}^{n} c_m(t)\xi^m \tag{20}$$

By considering only the quadratic solution in ξ and satisfying the boundary conditions of Eq. (8), one finds

$$\theta(\xi,t) = \xi^2 + g(t)(\xi - \xi^2) \tag{21}$$

where $g(t)$ represents the time-dependent coefficient. At $t = 0$, a combination of Eqs. (8) and (21) yields the result

$$g(0) = (1 - \xi^2)/(\xi - \xi^2) \tag{22}$$

Equations (21) and (22) are used to obtain specific solutions of Eqs. (9) and (18). Once the temperature distribution is known, the expressions for heat flux are obtained by numerically integrating the corresponding equations. The entire numerical procedure is described in detail in Refs. 24 and 25. The numerical procedure is similar for higher-order solutions in ξ of Eq. (20), but the computational resources required are considerably higher. The higher-order polynomials did not improve the solutions significantly for the physical problem considered in this study.

Physical Conditions and Data Source

For the physical problem considered (Fig. 1), four specific absorbing-emitting species were selected for an extensive study; these are CO, CO_2, OH, and H_2O. The species CO was selected because it contains only one fundamental vibration-rotation (VR) band and all spectral information is easily available in the literature. It is a very convenient gas to test the numerical procedure without requiring excessive computational resources. Species OH and H_2O are the primary radiation participating species for the pressure and temperature range anticipated in the combustor of the scramjet engine. Species CO_2 and combinations of CO_2 and H_2O are important absorbing-emitting species in many other combustion processes.

In radiative transfer analyses, it is essential to employ a suitable model to represent the absorption-emission characteristics of specific species under investigation. Several line-by-line, narrow-band, and wide-band models are available to model the absorption of a VR band (Refs. 7–11). However, it is often desirable to use a simple correlation to represent the total absorption of a wide band. Several such correlations are available in the literature (Refs. 7–11). The relative merits of these correlations are discussed in Ref. 11. In this study, the continuous correlation proposed by Tien and Lowder[7] is employed. This correlation is relatively simple and provides accurate results for pressures higher than 0.5 atm.

The spectral information and correlation quantities needed for CO, CO_2, and H_2O were obtained from Refs. 7–9. The spectral data for OH are developed in Ref. 25. The specific VR bands considered for each species are CO (4.7μ fundamental), OH (2.8μ fundamental), CO_2 (15, 4.3, and 2.7μ), and H_2O (20, 6.3, 2.7, 1.87, and 1.38μ). The radiative properties of important species are provided in Ref. 25.

For the specific problem considered, the dependent variables are θ, Q, and $\bar{Q}$, and independent variables are ξ and t. The parameters for general solutions are T_1, T_2/T_1, u_o, and t_m. For the radiative equilibrium case, θ and Q depend only on t and N_1 in the optically thin limit and on ξ, t, and M_1 in the large path length limit. For the case of combined radiation and conduction, θ and $\bar{Q}$ depend on ξ, t, and $N = N_1/R$ in the optically thin limit and on ξ, t, and $M = M_1/R$ in the large path length limit. Information on the radiative ability of various species in the optically thin and large path length limits is available in Refs. 8 and 25. The parameters for specific solutions for different species are T_1, T_2/T_1, P, L, and t_m. Extensive results, therefore, can be obtained by varying these parameters. For parametric studies, however, only certain values of pressure, temperature, and plate spacing were selected, and results were obtained for the general as well as limiting cases. Unless stated otherwise, specific results were obtained for $T_2 = 2\ T_1$ and for a characteristic time of $t_m = 0.00001$ s (Refs. 25 and 26).

Results and Discussion

Results have been obtained for different radiation participating species for both cases, the radiative equilibrium and radiation with conduction. Selected results are presented here, and extensive results are provided in Refs. 25 and 26. It should be realized at the outset that, according to the physics of the problem, the gas is initially at a high temperature, $T_1 = T_2$. At $t = 0$, the temperature T_1 is lowered to a constant value. The energy exchange then occurs, and the gas cools down in time until a steady-state condition is reached. At this time, a certain temperature profile is established, and a fixed rate of energy exchange occurs irrespective of the time. The rate of cooling of the gas layer, therefore, is dependent on the nature of the participating species and on the physical parameters of the problem.

Some limiting solutions that are independent of any participating species are presented first in Fig. 2 for the radiative equilibrium case. The temperature distribution in the channel is plotted as a function of the optically thin parameter N_1 for different times. These results show an exponential decay with time reaching the steady-state value of $\theta = 1/2$ for $t \to \infty$. The temperature distribution for the large path length limit is shown in Fig. 2 (with broken lines) as a function of the large path length parameter M_1 and for different times. Although the numerical values are entirely different, these results also show the exponential decay with time and reach the limiting value of $\theta = 1/2$ for $t \to \infty$. It should be noted that while the optically thin solutions are independent of the ξ coordinate, the large path length solutions do depend on ξ, and they have been obtained for $\xi = 0.5$. In the case of simultaneous radiation and conduction, both optically thin and large path length solutions for temperature distribution depend on ξ. These results, however, can be expressed in terms of the radiation-conduction parameter $N = N_1/R$ in the optically thin limit and $M = M_1/R$ in the large path length limit.

The results for the centerline temperature variations with time are compared for CO, OH, H_2O, and CO_2 in Fig. 3 for $P = 1$ atm, $T_w = 500$ K, and $L = 10$ cm. It is seen that H_2O is most effective and OH is least effective in transferring the radiative energy. The ability of a gas to transfer radiative energy depends on the molecular structure of the gas, band intensities, and physical conditions of the problem. Thus, H_2O with five strong VR bands is a highly radiation participating species, and the steady-state conditions were reached more quickly for H_2O than for other species. However, CO with one fundamental

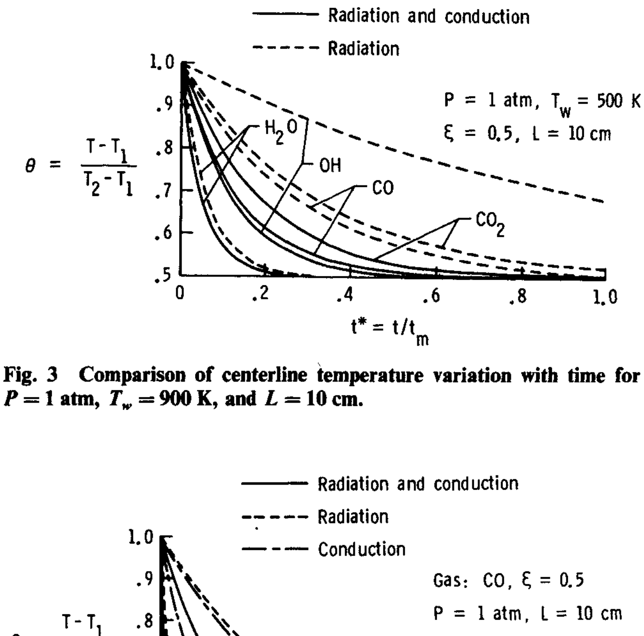

Fig. 3 Comparison of centerline temperature variation with time for $P = 1$ atm, $T_w = 900$ K, and $L = 10$ cm.

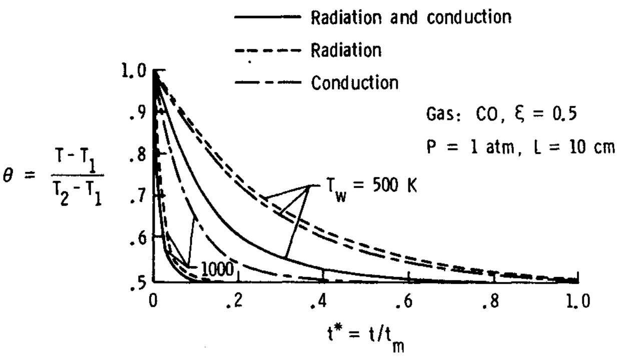

Fig. 4 Centerline temperature variation with time for CO with $P = 1$ atm and $L = 10$ cm.

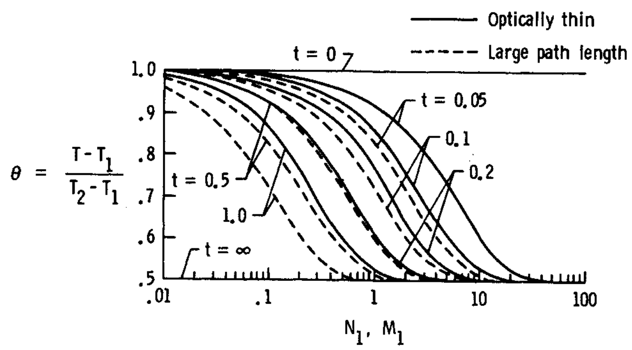

Fig. 2 Limiting solutions for the radiative equilibrium case.

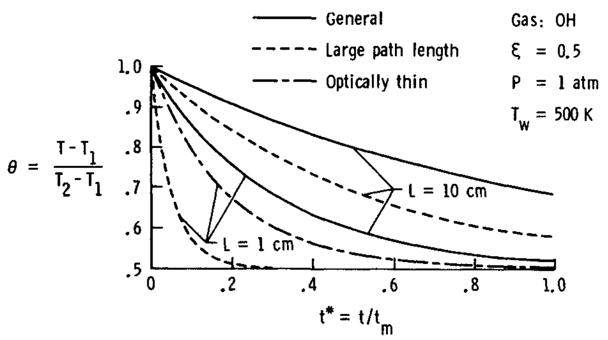

Fig. 5 Centerline temperature results for pure radiation (OH, $P = 1$ atm, and $T_w = 500$ K).

band is seen to be a better radiating gas than CO_2 with three VR bands. This is because for the given physical conditions the optical thickness of CO_2 is sufficiently large, and in the large path length limit CO_2 is relatively less effective in transferring the radiative energy (Ref. 8). Further results for CO are illustrated in Fig. 4 for different wall temperatures. It is seen that while radiation is less effective than conduction at $T_w = 500$ K, it is highly effective at $T_w = 1000$ K. This, however, would be expected because radiation becomes considerably important at higher temperatures. The steady-state condition is reached more quickly for $T_w = 1000$ K than for $T_w = 500$ K. In fact, for the characteristic time considered ($t_m = 0.00001$ s), the steady-state condition is reached more quickly for all species for temperatures higher than $T_w = 1000$ K (Refs. 25 and 26). Results for the pure radiation case are illustrated in Fig. 5 for OH for $L = 1$ and 10 cm. It is seen that while the general and large path length solutions depend on the plate spacing, the optically thin solutions are independent of the spacing. This fact was pointed out earlier in the method of solution. In the presence of other modes of energy transfer, the optically thin solutions also depend on the plate spacings. As would be expected, for the same physical conditions, the steady-state condition is reached faster for the lower plate spacing.

The temperature variations within the plates are shown in Figs. 6 and 7 for different species and for $P = 1$ atm and $L = 5$ cm. In the absence of molecular conduction, temperature jumps (radiation slips) occur at the boundaries and, therefore, the general solutions for the case of radiative equilibrium are not presented. However, general as well as limiting solutions are presented for the case of radiation with conduction. It is noted, in general, that for the case of radiation with conduction, the steady-state conditions are reached for all species at $t \geq 0.1$ and for $T_w \geq 500$ K. For the case of pure radiation, the steady-state conditions are reached at relatively longer times. The optically thin results are seen to be independent of the ξ coordinate for the case of radiative equilibrium and to vary slowly in the central portion of the plates for the case of radiation with conduction. This is because, in this limit, the gas interacts directly with the boundaries and conduction is predominant near the walls. Specific results for H_2O are illustrated in Fig. 6 for the case of radiation with conduction. These results demonstrate the typical trends for limiting and general solutions, i.e., a lower temperature gradient implies a higher rate of energy transfer. It is seen clearly that the rate of cooling is significantly higher in the large path length limit, and the steady-state conditions are reached at relatively longer times for lower T_w values (Refs. 25 and 26). For the case of combined radiation and conduction, a comparison of results for different species is shown in Fig. 7 for $t = 0.01$ and 0.1. The results for $t = 0.1$ essentially correspond to the steady-state conditions.

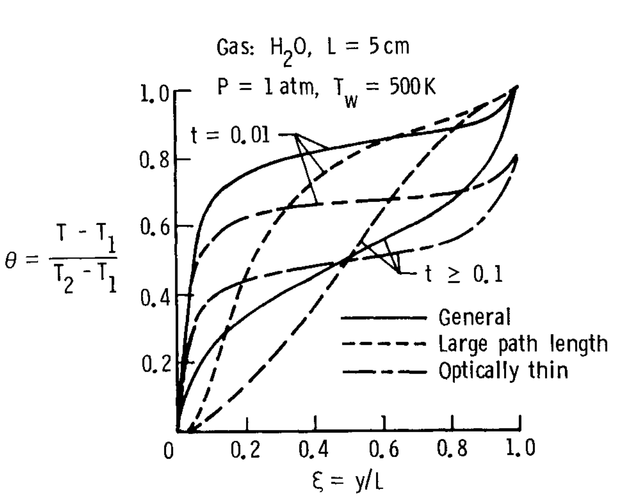

Fig. 6 Temperature variation for combined radiation and conduction for H_2O with $P = 1$ atm, $T_w = 500$ K, and $L = 5$ cm.

For $t = 0.01$, the variation in temperature is seen to be relatively small between $\xi = 0.2$ and 0.9. The centerline temperature is found to be the lowest for H_2O, and this is followed by OH, CO, and CO_2. However, it is noted that OH is very effective in transferring the net energy in comparison to the other species. This is mainly due to the relatively higher conductive ability of OH at $T_w = 500$ K.

The centerline temperature distributions are shown in Figs. 8–11 for different gases as a function of the spacing between the plates. In most figures, results are presented for both cases, the radiative equilibrium and radiation with conduction. For a particular gas, specific results are presented for various times to demonstrate the radiative nature of the gas under different pressure and temperature conditions.

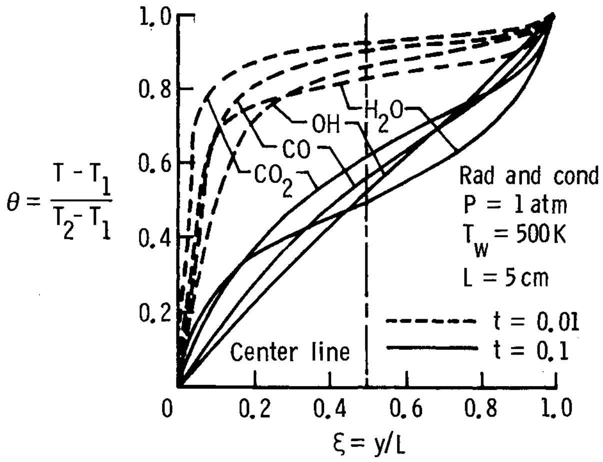

Fig. 7 Comparison of temperature variation for combined radiation and conduction for $P = 1$ atm, $T_w = 500$ K, and $L = 5$ cm.

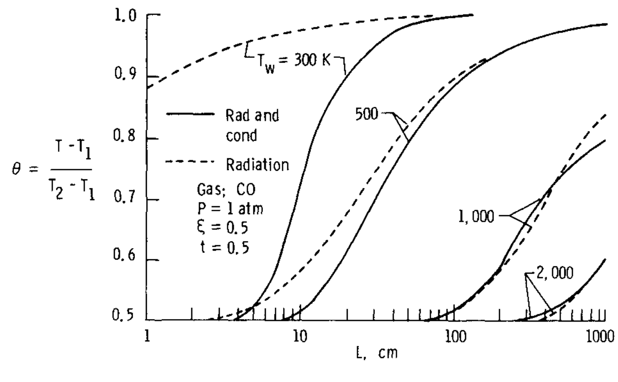

Fig. 8 Centerline temperature variation with L for CO ($P = 1$ atm and $t = 0.5$).

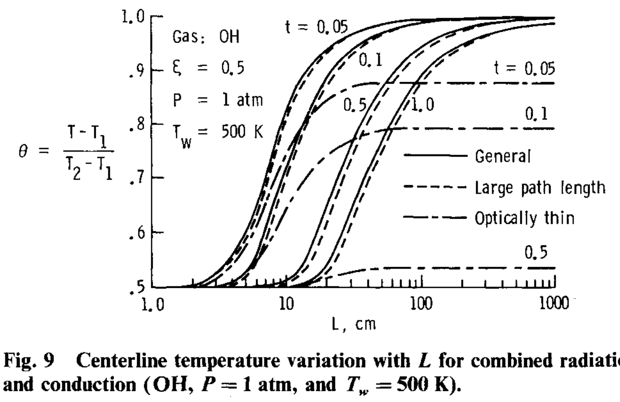

Fig. 9 Centerline temperature variation with L for combined radiation and conduction (OH, $P = 1$ atm, and $T_w = 500$ K).

The centerline temperature variations for CO are shown in Fig. 8 for $T = 0.5$, $P = 1$ atm, and different values of T_w. As would be expected, both conductive and radiative interactions increase with increasing temperatures, although the increase in radiative transfer is comparatively higher. It should be noted that for $T_w = 300$ K, $T_2 = 2$, $T_w = 600$ K, for $T_w = 500$ K, $T_2 = 1000$ K, and so on. Thus, for a higher value of $T_w = T_1$, the energy interactions occur at a sufficiently large temperature difference between the upper and lower plates. At these temperatures, if the plate spacing is small, the energy is transferred quickly and the steady-state condition is reached at relatively shorter times. This fact was also pointed out in the discussion of the results in Fig. 5.

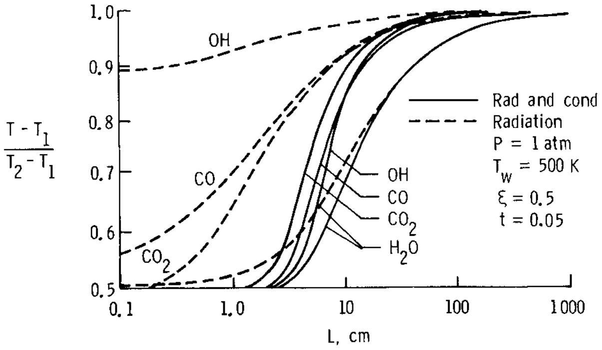

Fig. 10 Comparison of centerline temperature variation with L for $P = 1$ atm, $T_w = 500$ K, and $t = 0.05$.

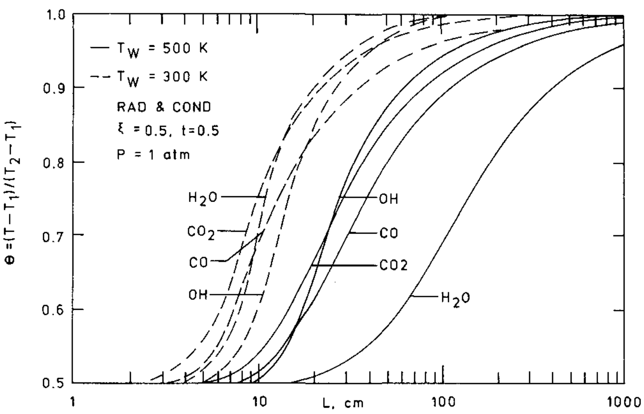

Fig. 11 Comparison of centerline temperature variation with L for combined radiation and conduction ($P = 1$ atm and $t = 0.5$).

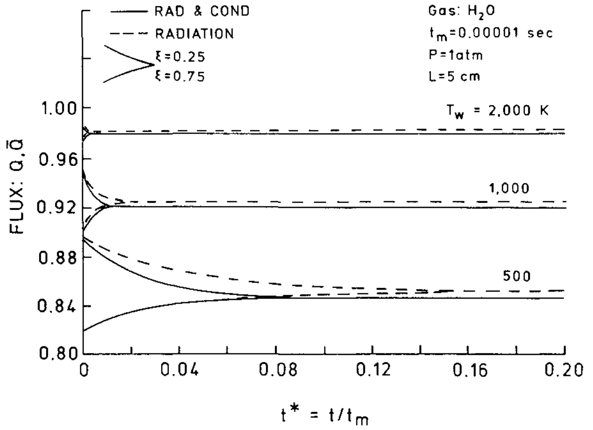

Fig. 12 Variation in heat flux with time for H_2O ($P = 1$ atm and $L = 5$ cm).

General and limiting solutions for radiative equilibrium are given in Refs. 25 and 26, and those for radiation with conduction are shown in Fig. 9b. These results clearly demonstrate the typical radiative interaction trends for different times. Once again, the results show that the optically thin solutions are independent of the plate spacing in the case of pure radiation but depend on the spacing when molecular conduction is included. The large path length results are seen to be valid only for large values of L for the case of pure radiation, but they appear to be valid in the entire range for the case of radiation with conduction.

Extensive results of $\theta(\xi = 0.5)$ vs L have been obtained for H_2O and CO_2 for different conditions, and these are discussed in Refs. 25 and 26. In general, these results show similar trends as exhibited by the results for CO and OH, but the extent of radiative interactions is entirely different.

The centerline temperature variations are compared for different gases in Fig. 10 for $P = 1$ atm, $T_w = 500$ K, and $t = 0.05$. For the case of radiative equilibrium, it is noted that OH is the least effective and H_2O is the most effective gas in transferring the radiative energy for plate spacings greater than 2 cm. When molecular conduction is included, OH becomes more effective because of its relatively higher conductive ability. These points were also noted in earlier discussions. The story, however, can be entirely different for other physical conditions because of the radiative/conductive nature of participating species (Refs. 8, 25, and 26). This fact is partially evident from the steady-state results for the case of combined radiation and conduction presented in Fig. 11 for two different temperatures, $T_w = 300$ and 500 K. For example, for $T_w =$ 300 K and $L = 10$ cm, the temperature values for CO and CO_2 are about the same, for H_2O it is lower, and for OH it is the lowest; however, for plate spacing greater than $L = 20$ cm, the trend is entirely different. Also, it should be noted that the steady-state ($t = 0.5$) results for $T_w = 500$ K in Fig. 11 show a different trend than do the results for the same temperature in Fig. 10 for $t = 0.05$. Thus, in order to predict the relative ability of a gas for radiative interactions, it is very important to consider the exact physical conditions for all species. These predictions may not be applicable if physical conditions of the problem are changed.

Extensive results for the variation in heat transfer can be presented analogously to the variation of temperature for different conditions. However, this should not be necessary, because the heat-transfer variation follows the trend of the temperature variation in a reverse manner. If the temperature differences are higher, the rate of heat transfer will be higher and the steady-state conditions will be reached earlier. The results for heat-transfer variations have been obtained for selected conditions and are available in Refs. 25 and 26. Only the results for H_2O are presented here to show the trend in cooling rates for different wall temperatures.

For $P = 1$ atm, the results for Q and $\bar{Q}$ are illustrated in Fig. 12 as a function of t^*. As would be expected, the results show that for a given plate spacing, the gas layer reaches the steady-state condition faster at higher values of T_w because of stronger radiative interactions. It should be noted that the rate of energy transfer increases with time for a gas layer closer to the upper wall ($\xi = 0.75$) and decreases with time for a gas layer closer to the lower wall ($\xi = 0.25$) until the steady-state conditions are reached. The rate of cooling is entirely different if the plate spacing is changed (Refs. 25 and 26).

Conclusions

The problem of transient radiative interaction in nongray absorbing emitting species has been formulated in a general sense such that sophisticated absorption models can be used to obtain accurate results if desired. Results have been obtained for the special case of radiative interactions in a plane gas layer bounded by two parallel plates when the temperature of the bottom plate is suddenly reduced to a lower but constant temperature. The energy transfer by pure radiation and by simultaneous radiation and conduction were considered, and specific results have been obtained for CO, CO_2, H_2O, and OH by employing Tien and Lowder's correlation for band absorption. It is noted that the extent of radiative interaction is dependent on the nature of the participating species and parameters T_1, T_2/T_1, P, L, and t_m. The steady-state conditions are reached at relatively longer times for radiative equilibrium than for radiation with conduction. For a particular value of P and T_1, the time required to reach the steady-state condition increases with increasing plate spacing. For a fixed plate spacing, the energy is transferred quickly for higher T_1 values because of large temperature differences between the plates. The rate of radiative interaction increases with increasing pressure until the large path length limit is reached. The radiative equilibrium solutions are found to be independent of the plate spacing in the optically thin limit. In the case of simultaneous radiation and conduction, both optically thin and large path length solutions depend on the y location between the plates. At moderate temperatures, OH is a poor radiating but better heat-conducting gas. For most conditions, H_2O is found to be a highly radiation participating species, and the steady-state conditions are reached more quickly for H_2O than for other species.

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